Problem I: *(10 points)* 9

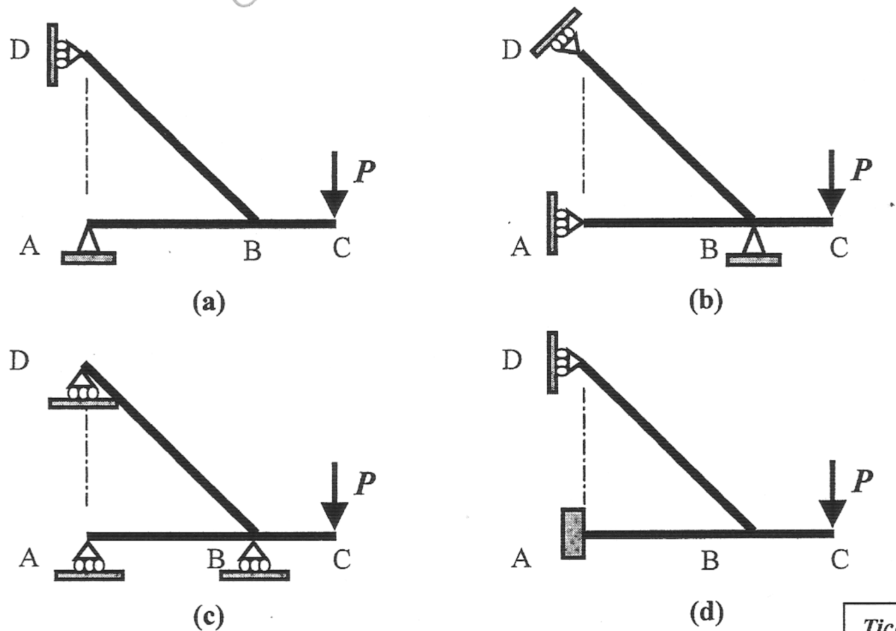


Figure I

Tick Boxes to check that you solved all questions ☑

Discuss very briefly the EXTERNAL stability and determinacy of each of the structural systems shown in Figure I (maximum of 2 lines each). *(10 points)*

Calculations and/or Diagrams:

2

(a) 3 equations and 3 unknowns, the system is stable and statically $\boxed{\text{determinate}}$ ✓ 2

(b) We have 4 unknowns and 3 equations $\boxed{\text{Statically indeterminate}}$ and unstable due to concurrent reactions 2

(c) Even though we have 3 unknowns and 3 equations, we have a $\boxed{\text{stability problem}}$. Conditional stability, if we apply a force that has an horizontal component, the system will move horizontally. 1

(d) 4 eq > 3 unknowns the fixed support at A prevents rotation and any horizontal or vertical displacement (stable) → $\boxed{\text{Statically indeterminate}}$ 2

Problem II: *(15 points)* 15

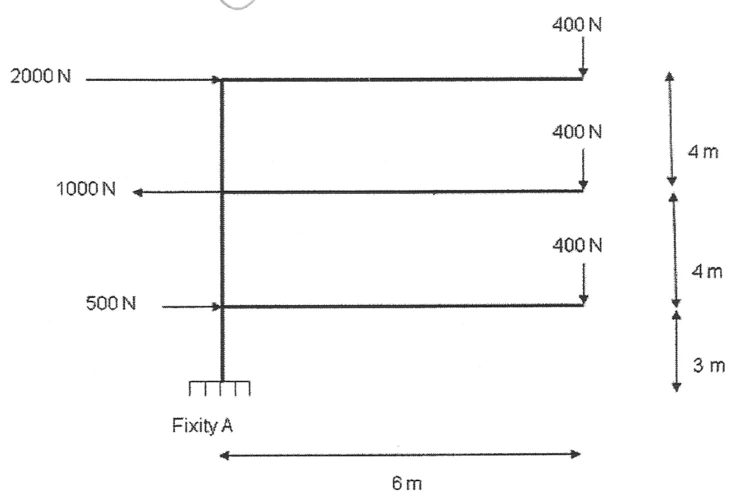
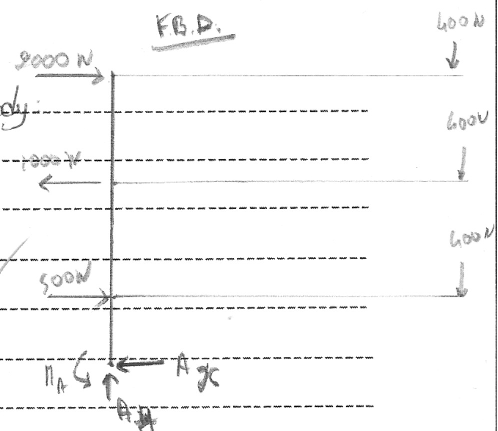


Figure II

Figure II shows a section of a building with 3 floors. Only the balcony section is shown for simplicity. There are 6 forces acting on the building as shown. If the building is in equilibrium, determine the reactions at foundation level (Fixity A). *(15 points)*

Calculations and/or Diagrams:



For the equilibrium of a rigid body:

$\xrightarrow{+} \Sigma F_x = 0$ $+\uparrow \Sigma F_y = 0$ $+\circlearrowleft \Sigma M_A = 0$

* $\xrightarrow{+} \Sigma F_x = 0$ $2000 - 1000 + 500 - A_x = 0$

$A_x = 2000 - 1000 + 500$

$\boxed{A_x = 1500\ N}$ ← A_x

* $+\uparrow \Sigma F_y = 0$ $-400 - 400 - 400 + A_y = 0$

$A_y = 3 \times 400$

$\boxed{A_y = 1200\ N}$ $\uparrow A_y$

* $+\circlearrowleft \Sigma M_A = 0$ $-400 \times 6 - 400 \times 6 - 400 \times 6 - 2000 \times 11 + 1000 \times 7 - 500 \times 3 + M_A = 0$

$-23700 + M_A = 0$

$\boxed{M_A = 23700\ N.m}$ $\circlearrowleft M_A$

Problem III: *(30 points)* 24

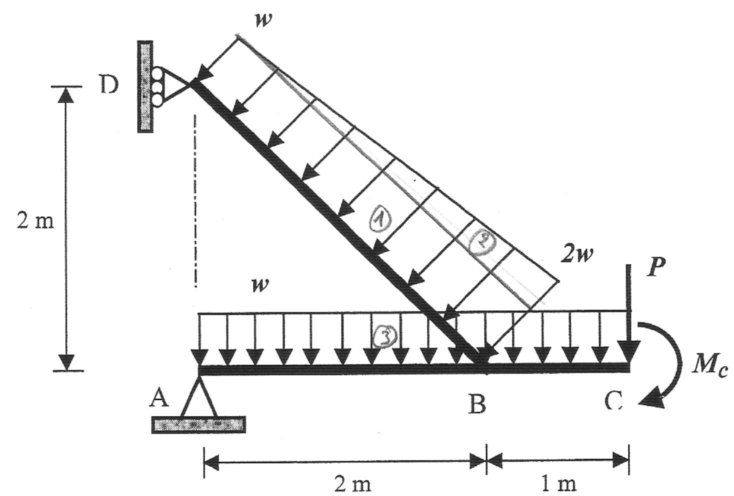


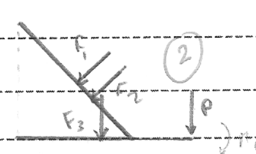
Figure III

The structural system shown in Figure III is stable and statically determinate.
Let w=500 N/m, P=1,000 N, and M_C=2,000 Nm.

1- Find a single resultant force (magnitude and direction) that is equivalent to the loads applied. *(15 points)*
2- Determine the location of this force on AC. *(8 points)*
3- Deduce the reactions at A and D. *(7 points)*

Calculations and/or Diagrams:

1. (1) $F_1 = w \times BD = 1414\ N$ (2) $BD = \sqrt{2^2+2^2} = 2.82\ m$
applied perpendicularly to member BD at it's middle.
(2) $F_2 = \frac{1}{2}\ BD \times w = 707\ N$ ✓ (2)
applied perpendicularly to member BD at a distance
$x_2 = \frac{2}{3} BD = 1.88\ m$ from D.
(3) $F_3 = AC \times w = 3 \times w = 1500$ ✓ (2)
applied perpendicularly to members AB and BC
at a distance $x_3 = 1.5\ m$ from A.



(2)

forces F_1 and F_2 make an angle of 45° with the horizontal line.

Calculations and/or Diagrams (cont'd):

$\vec{F_R} = \vec{F_1} + \vec{F_2} + \vec{F_3}$

$\overset{+}{\rightarrow}$ $F_{R_x} = -F_1 \cos 45 - F_2 \cos 45 + 0$

$F_{R_x} = -1499.8\,N$ ✓ (2)

(1)

$+\uparrow$ $F_{R_y} = -F_1 \sin 45 - F_2 \sin 45 - F_3$ $\rightarrow P$

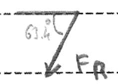
$F_{R_y} = -2999.8\,N \rightarrow 4000$

magnitude: $|\vec{F_R}| = \sqrt{F_{R_x}^2 + F_{R_y}^2}$

$\boxed{F_R = 3353.8\,N}$ (2)

direction: let θ be the angle between F_R and a horizontal line: $\tan\theta = \frac{F_{R_y}}{F_{R_x}}$

$\boxed{\theta = 63.4°}$



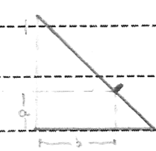
2. The moments of forces F_1, F_2 and F_3 on A is equal to the moment of F_R: (F_1 passes though A so its moment is 0)

$+↺$ $-\bar{x}\, F_R \sin\theta = F_2 \cos 45 \times a - F_2 \cos 45 \times b - F_3 \times 1.5$ $+ M_C$

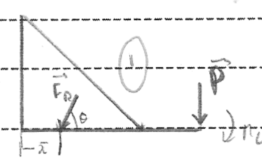
(1)

$\frac{a}{2} = \frac{(2.82 - 1.88)}{2.82}$ $a = 0.67$

$\frac{b}{2} = \frac{1.88}{2.82}$ $b = 1.33$

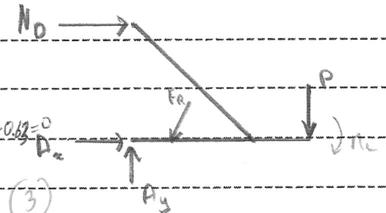


$\boxed{\bar{x} = 0.63\,m}$ (2)



Calculations and/or Diagrams (cont'd):

3. Support reactions.

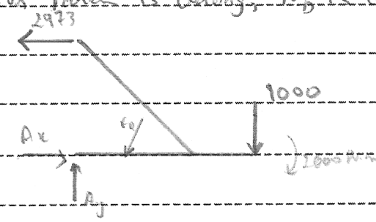


$+\circlearrowleft \Sigma M_{@A} = 0$

$-N_D \times 2 - P \times 3 - M_C - F_R \cos 63.4 \times 0.63 = 0$

$\boxed{N_D = -3444.6\,N}$ (3)

Therefore the assumption taken is wrong, N_D is directed to the left



$+\uparrow \Sigma F_y = 0 \quad A_y - 1000 - F_R \sin 63.4 = 0$

$\boxed{A_y = 3998.8\,N}$ ✓ (2)

$\xrightarrow{+} \Sigma F_x = 0 \quad A_x - N_D - F_R \cos 63.4 = 0$

$\boxed{A_x = 4946.3\,N}$ (2)

Problem IV: *(30 points)* 30

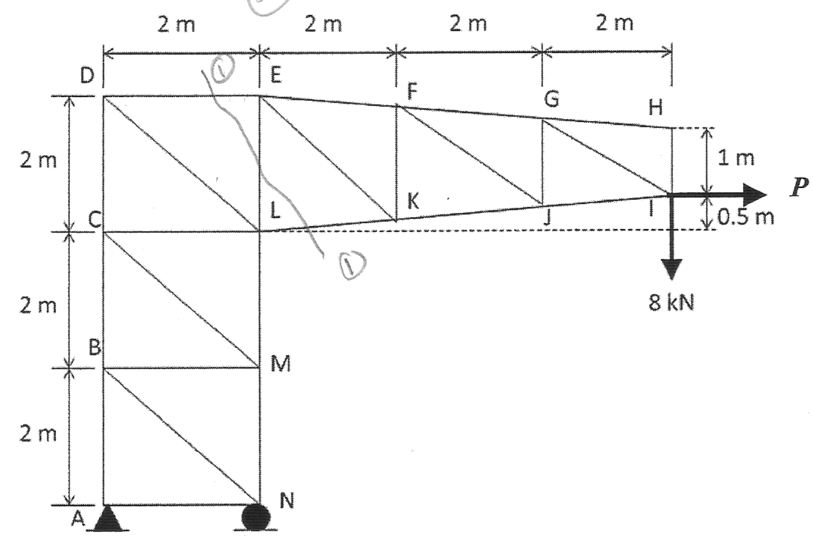


Figure IV

The plane truss shown in Figure IV is stable and statically determinate externally and internally.

1- Confirm the determinacy. *(3 points)*
2- Let P=0.
   Use the method of sections to compute the internal axial forces in members DE, EL, and LK, for the 8 kN load shown, and determine whether these members are in tension or compression. *(15 points)*
   Deduce the forces in members DL and DC using one appropriate joint. *(5 points)*
3- Calculate the maximum horizontal force P that can be applied at joint I to the right (in addition to the existing 8 kN vertical load) so that member DE does not fail. Assume that the maximum load in tension that the truss members can take is 26 kN, while the maximum compression force is 38 kN. *(7 points)*

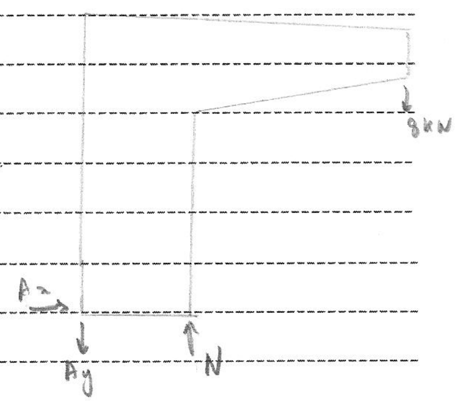
Calculations and/or Diagrams:

1. members $m = 25$
reaction $r = 3$
joints $j = 14$
$m + r = 28$ $\quad 2j = 28$ $\quad$ 3
$m + r = 2j$
We are not in the case where reactions are concurrent or parallel, hence the truss is stable and statically determinate

Calculations and/or Diagrams (cont'd):

2. support reactions:

for P = 0



$+\circlearrowleft \sum M_{@A} = 0$

$-8 \times 8 + N \times 2 = 0$

$\boxed{N = 32 \text{ kN}}$

$+\uparrow \sum F_y = 0$

$-A_y + N - 8 = 0$

$A_y = N - 8$

$\boxed{A_y = 24 \text{ kN}}$

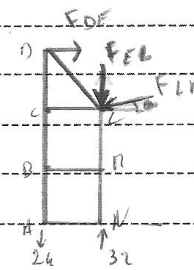
$\xrightarrow{+} \sum F_x = 0$

$\boxed{A_x = 0}$

Using the method of sections: Section 1-1

$\tan\theta = \frac{0.5}{6}$

$\theta = 4.76°$



1

$+\circlearrowleft \sum M_{@L} = 0$

$24 \times 2 - F_{DE} \times 2 = 0$ 2

$\boxed{F_{DE} = 24 \text{ kN}}$

Tension

1

$\xrightarrow{+} \sum F_x = 0$

$F_{DE} - F_{LK} \cos\theta = 0$

$F_{LK} = \frac{F_{DE}}{\cos 4.76}$ 2

$\boxed{F_{LK} = 24.08 \text{ kN}}$ Compression

1

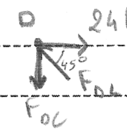
$+\uparrow \sum F_y = 0$

$-24 + 32 - F_{EL} - F_{LK} \sin 4.76 = 0$

$\boxed{F_{EL} = 6 \text{ kN}}$ 2 Compression

Calculations and/or Diagrams (cont'd):

Using the method of joints for joint D:



$\xrightarrow{+} \sum F_x = 0$

$24 - F_{DL} \cos 45 = 0$

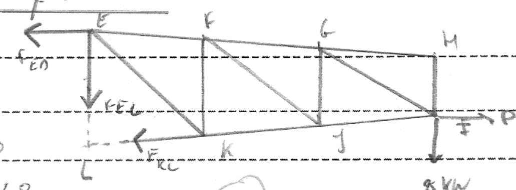
$\boxed{F_{DL} = 33.9\ kN}$ Compression

$+\uparrow \sum F_y = 0$

$F_{DL} \sin 45 - F_{DC} = 0$

$\boxed{F_{DC} = 24\ kN}$ Tension

3. We use the method of sections:



$\circlearrowleft \sum M_{@L} = 0$

$F_{ED} \times 2 - P \times 0.5 - 8 \times 6 = 0$

$F_{ED} = \dfrac{0.5P + 48}{2}$

$F_{ED} > 0$ is in tension $F_{ED_{max}} = 26\ kN$

$\dfrac{0.5P + 48}{2} = 26$ $P = \dfrac{2 \times 26 - 48}{0.5}$

$\boxed{P = 8\ kN}$

Problem V: *(15 points)*

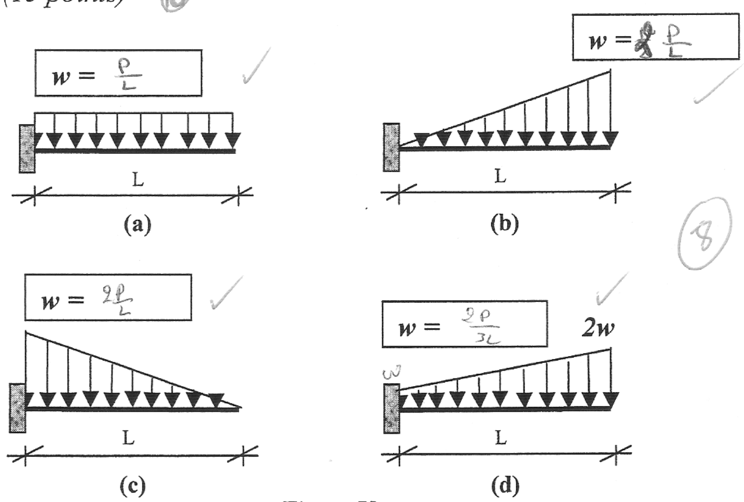


Figure V

The fixed-end cantilever beams shown in Figure V are loaded with different types of distributed loads in (a), (b), (c), and (d).

1- If the equivalent load on each of these beams is the same and equal to P, determine the value of w for each the cases and write it on the figures above. *(8 points)*

2- Knowing therefore that the vertical reactions at the fixed ends are equal to P upward in all beams, estimate, without calculations, which beam has the largest (counter-clockwise) moment reaction to smallest; write them down in the order as below and explain the reason of your choice VERY briefly in 2-3 lines maximum. *(7 points)*

$$M(c) > M(a) > M(d) > M(b)$$

b d a c

Calculations and/or Diagrams:

1 (a) $P = wL$ $w = \frac{P}{L}$

(b) $P = \frac{1}{2}wL$ $w = \frac{2P}{L}$

(c) $P = \frac{1}{2}wL$ $w = \frac{2P}{L}$

(d) $P = P_1 + P_2 = wL + \frac{1}{2}wL$

$= w\left(\frac{3L}{2}\right)$

$w = \frac{2P}{3L}$

2. $\Sigma M = 0$

$M_{Reaction} = P \times d$ where d is the distance between the support and the point of application of the force

$d_a = \frac{L}{2}$ $d_b = \frac{2L}{3}$ $d_c = \frac{L}{3}$ $\frac{L}{2} < d_d < \frac{2L}{3}$